How we know the Earth moves

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August 28, 2009

Introduction

In October, 1990, I was teaching a class in the history of mathematics to students who were mostly future high school teachers of mathematics. I assigned the class to debate the question raised by Galileo, *Resolved: that the Earth moves.* They were to advance arguments pro and con and refute the arguments raised by the other team. When the debate was held, neither side made a single coherent argument, and in a very Californian fashion, they agreed that it didn't matter anyway, and was merely a matter of opinion: one could choose to regard the Earth as stationary, and another could choose to regard it as moving, and both could be right. I told them that it is *not* a matter of opinion, but a matter of *evidence*, and sent them packing to the library (this was in the Dark Ages before the Web came into being). The next week, the debate was held again, at a somewhat more acceptable level. Still, very few people can actually give a coherent answer to the question, "how do we know that the Earth moves?". Therefore, in this short note I will set out what I know about this question.

There are several kinds of motion of the earth: its daily rotation, its yearly revolution about the sun, the variations in its orbit (changes in ellipticity, precession, etc.), and finally the motion of the whole solar system relative to the galaxy and the motion of the galaxy relative to other galaxies. There are different answers to how we know about these different kinds of motion, and several interesting stories about the history.

1 Galileo and Copernicus

2 Foucault's pendulum

If you sit opposite someone on a merry-go-round and try to play catch, you'll find it difficult, because by the time the ball gets to them, they will have moved. Similarly, if you set up a pendulum at the North Pole, it would move in one (vertical) plane while the earth turned under it. What you would see is that the pendulum would appear *not* to move in a plane, but that the "plane" of its swing would turn in a circule once a day. If the pendulum is not at one of the poles, it is more complicated to analyze what it should do; but what it *doesn't* do is stay in one plane, as it should if the Earth were stationary. One can find the calculation easily following links from the Wikipedia article on the Foucault pendulum, so there is no point in giving it here.

The first such pendulum was constructed by Joseph Foucault in 1851, for the World's Fair in Paris. It was suspended by a wire 200 feet long, and traced its path in a thin layer of sand. It was started by pulling it to one side with a thread and then burning through the thread with a match, to avoid any lateral impulse.

There is Foucault pendulum on display at the Academy of Science in Golden Gate Park, San Francisco.

Foucault's pendulum proves the rotation of the Earth. If you could keep a pendulum going for a year, or significant fraction of a year, you might prove the revolution too, because in a year you get one extra rotation of the pendulum relative to the earth. At least, you would get exactly one extra rotation if the pendulum were at the North Pole and the axis of the Earth were perpendicular to the ecliptic. (The "ecliptic" is the plane of the Earth's orbit around the sun.) This "thought-experiment" is useful to refute the argument that it is all a matter of viewpoint and you can choose for yourself whether to regard the earth as moving or not.

Foucault published his work in 1851 [3]. This paper has been translated and posted to the web, so you can see for yourself that Foucault clearly understood the principles involved and also the relation to Coriolis force (discussed below), and he credits the mathematician Poisson with a correct analysis of that phenomenon.

One year later, in 1852, Foucault learned about a new invention, the gyroscope, and realized that a gyroscope could be used to demonstrate the Earth's rotation, without the complications of the calculation of what should happen at a latitude other than the North Pole. The gyroscope would keep pointing to the same point in the heavens for 24 hours while the Earth turns. For example, if we point the axis of rotation straight up, and we are not at either the North or South Pole, then in the course of 24 hours the gyroscope's axis of rotation will appear to trace out a curve, returning to its starting position after about a day. However, Foucault's experiment was unsuccessful, as his gyroscope could not be kept running longer than ten minutes. Not until electric motors could be used to keep the gyroscope turning could a gyroscope be used for either a demonstration of the Earth's rotation, or more practically, for marine navigation. Practical gyroscopes emerged in 1905–1910.

Actually, since the Earth moves around the Sun as well as around the Earth, it should return to its starting position in slightly less than one day; thus the gyroscope, or the Foucault pendulum, demonstrates that the Earth not only rotates relative to the fixed stars, but also has other motions as well; and measuring the exact time of return to the starting position provides evidence that the Earth moves around the sun once a year.

It is worth noting that, while an unmotorized gyroscope cannot run very long, a Foucault pendulum usually is also powered, but in that case, a clock-like mechanism can be used to give the pendulum an extra impulse at each swing.

In any case, it is not necessary that the pendulum or gyroscope actually run for twenty-four hours to demonstrate that the Earth moves–it only has to run long enough to verify that the plane of oscillation is changing relative to the Earth. Half an hour is more than enough.

Question: Was metallurgical technology able to construct a Foucault pendulum or a gyroscope in the time of Copernicus (1543) if anyone had thought of it?

3 Coriolis effect

The same principle can be seen in the flight paths of rockets, although you can't measure this for yourself unless you have an ICBM or a naval gun. If you were to fire a rocket due north, over the pole for example, by the time it landed, the original target would have moved eastward due to the Earth's rotation. You could see that you had aimed right, because your rocket would pass over the pole. The same effect can be seen in a shorter flight, and has to be taken into account when aiming long-range artillery such as naval guns.

If you try this with an airplane, it won't work, or at least not so well, because the Earth's atmosphere carries your airplane along with it. The rocket escapes the "wind" of the atmosphere and so continues in the direction "north" was at the time of launch; hence it will soon appear to be in the northwest as viewed from the launch pad; the bullet goes so fast that the wind doesn't have time to work. But the effect can be observed in a airplane with a level bubble. Say the plane was flying north: the bubble appears to experience a "force" to the east. It's actually just the water in the level trying to continue on a straight path, while the airplane is being turned from a straight path to keep flying on a line of longitude. Navigators in the pre-digital age had to compensate for this effect because some of their instruments used bubbles.

Similarly, rivers flowing north or south scour out their west banks (in the northern hemisphere) more than their east banks, and this effect is more pronounced at polar latitudes. It has been observed, for example, in the Yukon River. The effect is also said to influence the large-scale weather patterns in the northern and southern hemispheres.

The effect is sometimes called the "Coriolis force". Like "centrifugal force", it is not an actual force, but an effect of circular motion that may feel like a force to a person on the rotating body. If you try to walk radially on a merry-go-round, you will find that you have to lean over and push sideways, in order to maintain a radial motion. The "force" you are resisting is the Coriolis force.

Coriolis published his analysis of the effect in 1834, some years before Foucault constructed his pendulum. Did Foucault read Coriolis or make an independent discovery?

4 Seasons

Consider that the northern and southern hemispheres have opposite seasons, and the poles have a 24-hour daylight period in the summer and 24-hour darkness in the winter. This is not possible if the Earth is stationary and day and night are caused by the sun orbiting the Earth in a fixed plane passing through the center of the Earth, for then day and night would always be 12 hours long. To explain the seasons with the sun going around the Earth, we would have to assume that the plane of the sun's orbit changes with the seasons, so the sun is higher in the sky in the summer and lower in the winter. What would cause that effect? Of course, before the time of Galileo, the laws governing motion were unknown, so why shouldn't an orbit change planes? By themselves, the seasons don't prove that the Earth moves; they do that only when supplemented by Newton's laws of physics. But as soon as we believe that some force is required to change an orbit, it would be hard to account for the seasons with a stationary Earth.

5 Diurnal rising and setting of the stars

This phenomenon was noticed by the ancients in connection with their efforts to regulate the calendar. As Copernicus pointed out, if we assume the Sun is stationary and the Earth revolves around it, these "risings and settings of the starts, whereby they become morning and evening stars", can easily be explained. To explain them with the Earth stationary and unmoving, we must assume that the whole sphere of fixed stars rotates about the Earth once a year in addition to once a day, that is, 366 times a year instead of 365. Why the period of the fixed stars should be exactly one day more than the period of the Sun is hard to explain with a stationary Earth.

This was probably the most convincing evidence for a moving Earth that was available during the lives of Copernicus and Galileo.

6 Motion of the Planets

The planets appear to change position relative to the "fixed stars". If the earth is stationary, and the planets move around it, then they certainly can't just move in circles, because they exhibit "retrograde motion": sometimes their motion against the fixed stars changes direction for a while, before reversing again and continuing in the original direction. Before Copernicus, this was explained by "epicycles." An epicycle is the path of a point moving in a circle whose center moves in another circle, i.e., a "circle on a circle". This theory of planetary motions, due to Ptolemy, also involved circles on circles on circles, and circles on circles on circles on circles, etc.

The necessity for epicycles is already enough to refute the "crystal sphere" hypothesis and raise the question, what holds the planets up? (You could still assume a crystal sphere to hold all the fixed stars up.)

Refinements of the epicycle theory were enough to make reasonably accurate predictions of the planet's motions.

Copernicus proposed that the planets and sun move around the center of the Earth's orbit, and that the Earth moved just like the planets. (His theory is often loosely described as saying that the planets and the Earth move around the sun, but that wasn't quite what he said.)

The two theories *both* could make numerical predictions, in principle the same ones. In that case, other factors such as simplicity of the theory or conflict with religious beliefs may influence one. But if two theories make the same predictions, one can't choose between them on scientific grounds. Hence, the motion of the planets is not, in itself, evidence for the motion of the Earth.

7 Stellar Parallax

If the Earth moves around the sun, then this motion should produce an apparent motion of the stars as seen from the Earth. To see this, imagine two stars, A and B, which are in line when viewed from the sun, with A farther away. Then when the Earth is at one side, A will appear to the left of B, and six months later, A will appear to the right of B. As the Earth circles the sun, A and B will appear to circle each other (if the line AB is not in the plane of the Earth's orbit). This effect is called "stellar parallax".

Copernicus anticipated this objection to his theory, and answered it by saying that the starts must be so far away that we can't notice the effect. Before you would even look for this effect, you would have to realize that perhaps not all the "fixed stars" are at the same distance from Earth. (This was not yet realized at the time of Galileo.)

To observe and measure the stellar parallax, one needs good optical instruments, because the effect is very small. In fact, the effort to measure the stellar parallax and so verify Copernicus gave considerable impetus to the development of the technology of optics, and the repeated failures to measure it didn't help the advance of the Copernican theory. It was finally measured successfully by Bessel in 1838 (for the star 61 Cygni, This measurement decisively proved that the Earth moves around the sun (relative to the fixed stars), and not the other way around. But it came two centuries after Galileo and Copernicus.

The angle in question is small. The largest stellar parallax ever measured is that of Proxima Centauri, and angle of 0.783 seconds of arc, about the angle subtended by a dime at a distance of three miles.

The measurement of stellar parallax is an important one, because it is the key to determining the scale of the universe. Using the red shift (explained below), we can determine the relative distances of different stars, but to calibrate this scale, we need to measure the actual distance of at least one star, and the only way to do that is by measuring the stellar parallax.

The English astronomer Thomas Digges attempted to measure the parallax of the 1572 supernova observed by Tycho Brahe. Failing to measure the parallax, he concluded that the supernova must be farther from Earth than the moon.

This led Digges to speculate that the stars were not attached to a celestial sphere (all at the same distance from Earth) but were instead scattered throughout three-dimensional space, which might contain infinitely many stars. See [2] Up until 1572 (well past the time of Newton) nobody seems to have questioned the idea that the stars were all at the same distance from the solar system. (Can this be true?)

Tycho Brahe, the careful experimental astronomer whose measurements Kepler used, never accepted the "heliocentric hypothesis" of Copernicus, because he knew that the stellar parallax was (if not zero) so small that under the heliocentric hypothesis, the stars would be unimaginably far away. The idea of such a large universe must have seemed revolutionary and frightening.

8 Aberration of Starlight

Suppose a telescope is moving perpendicular to the light entering it (or at any angle except directly towards or away from the light). Then by the time the light reaches the mirror or eyepiece, the telescope willhave moved slightly from where it was when the light entered the telescope. The result will be that the object seen appears to be shifted slightly from its actual position. This effect is called the "aberration of light." The velocity of the Earth's motion in its orbit about the sun should cause this effect when we look at stars not directly ahead or directly behind. Observing the same stars three or six months later, we should see them in a slightly different position. Stars in the plane of the Earth's orbit should simply appear to move back and forth. Stars not in that plane should appear to trace out small elliptical paths.

The effect is small: the most it can be is 20.47 seconds of arc. But note that this amount is still thirty times the stellar parallax, and it was measured a century sooner–but still a century after Copernicus and Galileo.

Here is a calculation of the maximum value of the aberration of starlight. Suppose the length of the telescope is L and the speed of light is c, and v is the velocity of Earth in its orbit. When the starlight is arriving perpendicular to the Earth's orbit, the time it takes for the light to traverse the telescope is $t = L/c$, and in that time the telescope moves $vt = Lv/c$. So the angle θ of aberration is given by $\sin\theta = (Lv/c)/L = v/c$. Somewhat surprisingly, the angle does not depend on the length L of the telescope.

Since $\sin\theta$ is very small, $\sin\theta$ will be almost exactly equal to θ, and we have $\theta = v/c$. The velocity v of Earth in its orbit is calculated, if we know the distance R from Earth to the sun, as $v = 2\pi R$ per year. Putting in modern values for R and c we have

$$\begin{aligned} R &= \text{150 million kilometers} \\ &= 1.5 \times 10^{11} \text{ meters per year} \end{aligned}$$

Now we use the useful fact that one year is approximately (to two decimal places accuracy) $\pi \times 10^7$ seconds.

$$
\begin{aligned}
v &= 2\pi R \text{ per year} \\
&= \frac{2\pi \times 1.5 \times 10^{11}}{\pi \times 10^7} \text{ meters per second} \\
&= 3 \times 10^4 \text{ meters per second} \\
c &= 3 \times 10^8 \ \text{ meters per second} \\
\theta &= \frac{v}{c} = \frac{3 \times 10^4}{3 \times 10^8} \\
&= 0.0001 \ \text{ radians} \\
&= 20.6 \ \text{ seconds of arc}
\end{aligned}
$$

This is slightly different from the value 20.49552 seconds of arc quoted in the Wikipedia article on "Aberration of Light" as the "accepted value". I do not know why.

This effect was first measured by James Bradley, the third Astronomer Royal of England, in 1725. See [1], pp. 258–265, and [4] for the fascinating history of this discovery. Bradley was looking for the stellar parallax, but instead discovered aberration. However, he could not explain his observations until 1727, so in this case, data preceded theory. Since at that time, the speed of light was not known, Bradley used the equations above "backwards", to proceed from the observed value of θ to the value of c, and this was the first determination of the speed of light. Bradley got the answer 183,000 miles per second, a bit shy of the true answer of 186,000 miles per second.

Question: why couldn't Bradley measure the stellar parallax, which is thirty times smaller?

9 Newton's Theory of Universal Gravitation

Isaac Newton said that every object ("body") in the universe attracts every other object, by a force F given by his famous law of universal gravitation,

$$F = \frac{GMm}{r^2}$$

in which M and m are the masses of the two objects, r is the distance between them, and G is a universal constant. Newton and many mathematicians who came after him, especially Laplace, were able to derive from this law (together with Newton's other law $F = ma$, where a is the acceleration of the body with mass m) many basic features of the solar system. In particular, that the planets move about the sun in elliptical orbits is a mathematical consequence of these two laws.

Kepler had suggested, and supported by accurate observations, that the orbits of the planets were elliptical, rather than given by epicycles (and epicycles on epicycles). To compose a perfect ellipse, you would need infinitely many epicycles on epicycles on epicycles, but as remarked above, to any given accuracy, enough epicycles will make the same predictions as ellipses. However, when Newton derived the elliptical-orbit theorem by pure mathematics from his simple law of gravitation, the beauty and simplicity of this theory, together with the power of the theory to make accurate predictions about the positions of the planets and their moons (without needing to assume more and more epicycles to get greater accuracy), provided a compelling argument in favor of the truth of Newton's theory. But while epicycles can be made to work with the assumption of a stationary Earth, Newton's theory requires the Earth, like every other body, to move in response to forces acting on it.

10 Motion of the solar system

At the beginning of the twentieth century, scientists (and everyone else) assumed that the universe was static, i.e., that the the stars did not move relative to each other or to the earth, or at least, did not move much. Also, the existence of anything beyond the Milky Way was not known. Today, we understand that the Milky Way is just one of millions of galaxies, but that was not discovered until 1929. In that year, Hubble discovered the *red shift.* The light from every star contains characteristic *spectral lines*, caused by the absorption of certain frequencies by hydrogen and helium (and other elements) present in stars. Hubble saw that these lines appear to be shifted towards lower frequencies (so visible light from the stars appears redder than it would if it had been omitted on Earth.) The simplest explanation of this is that all the stars are moving away from Earth! That does not necessarily place the Earth in the center of the universe, because if space itself is expanding like a balloon, every star is moving away from every other star.

If the amount of red shift represents velocity, then it seems reasonable that the most red-shifted objects are the farthest away. Hubble concluded that the distance of a star from Earth is proportional to its red shift; the constant of proportionality is the "Hubble constant" H. Hubble saw that the objects up until then known as "nebulae" were the most red-shifted of all, and concluded that they are immensely far away; hence they must also be immensely large. It was realized that they are galaxies, similar to the Milky Way in size.

Once it was realized that the Milky Way is "our galaxy", separated from other galaxies by large regions of empty space, then Newton's law of gravitation says that the Earth must be in an orbit around the galactic center. (Newton proved a famous theorem that says that the orbit in question will be the same as it would be if all the mass of the galaxy except that of the Earth were concentrated at the galactic center.) How can we detect the motion of the Earth relative to the rest of the galaxy?

The red shift answers this question as well. If in one direction of the sky, the stars are less red-shifted, and in the opposite direction, they are more red-shifted, and in directions halfway between they are all red-shifted by the same amount, halfway between the two extremes, then we must be moving in the first direction. The amount of the difference in red shifts can be used to compute our velocity. Once we have our velocity, however, our orbit is still not determined.

This section isn't finished yet.

References

[1] Berry, Arthur. *A Short History of Astronomy* (John Murray, 1898 republished by Dover, 1961).

[2] Wikipedia article on Thomas Digges.

[3] Foucault, M. L., Physical Demonstration of the Rotation of the Earth by Means of the Pendulum, *Comptes Rendus de l'Acad. de Sciences de Paris*, 3 Fevrier, 1851. Translation posted at `http://www.fi.edu/time/journey/Pendulum/foucault_paper_page_one.html`

[4] Hirschfeld, Alan. *Parallax:The Race to Measure the Cosmos*, Henry Holt, New York, New York (1985). ISBN 0-8050-7133-4.