The Meaning of Existence

Michael Beeson San José, August 2007

The Meaning of “Existence”

Michael Beeson San José, August 2007

“Does there exist a binary star at the heart of the Cat’s Eye Nebula?”

Wikipedia article

Reality and Truth in Mathematics

- That was the title of a 1998 paper of mine. But perhaps I bit off more than I could chew.
- Today we'll work on the Reality part (ontology), and leave Truth (epistemology) for another day.
- In 1998 I said that existence was *binary* and *fundamental* (unanalyzable, primitive).
- But today I will analyze it nevertheless.

Existence in Mathematics

- Greek geometry
- Cubic equations and complex numbers
- 19^{th} century and the notion of function
- Fourier series
- Dedekind and Cantor
- The paradoxes and avoiding them
- Zermelo and well-ordering the reals
- Brouwer and Hilbert
- The present situation

Existence in the history of science

- Atoms
- Gravity
- Phlogiston
- Heat
- Energy
- Fields
- The aether
- Quantum mechanics
- Quarks
- Virtual particles
- Quantum entanglement
- The “life force”

Existence in Philosophy

- Individual *vs.* general existence
- Four senses of “is”
- “Existence is not a property”--It is a second-order property, i.e. applies to concepts.
- Individuals and names
- Undefined terms, possible worlds
- Ontology *vs.* Epistemology
- Infinity

Reify

- To turn an adjective into a noun
- What we do to get “two” from “two-ness”.
- What we do to get “ideal numbers” from sets of rational numbers.
- What we do to get “a function” from its graph, or a rule for computing it.
- “to regard (something abstract) as a material or concrete thing”

--Merriam-Webster

Individual *vs.* General existence

- Dogs exist
- This dog “Penny” exists:

Is individual existence different from general existence?

- Russell said no. A name is a way of describing a property. "There is a dog with the property of being Penny".
- Or "..with the property of being the dog at which my finger is now pointed."
- Russell was right, in spite of the lingering doubts expressed in the *Oxford Dictionary of Philosophy* article on existence.
- A substantial fraction of the long *Stanford Encyclopedia of Philosophy* article on existence is devoted to this issue. There are 33 references.

Existence is not a property

- Properties (first-order ones) apply to objects, that is, to things that exist
- “there exists a dog” means “there exists something with the property of being a dog”
- Existence is a second-order property
- $\exists(\lambda x.\mathrm{dog}(x))$ using correct notation
- $\exists(\mathrm{dog})$ another way to say the same thing
- $\exists x.\mathrm{dog}(x)$ for short

Parametric existence *vs* Individual Existence

- Parametric existence involves the quantifier combination, "for every x there exists a y such that …"
- Example: For every polynomial ideal, there exists a finite basis.
- You might give a non-constructive existence proof, i.e. no *method* to find *y*, but still you might believe that for each actually exhibited *x*, you could find and exhibit a *y*.
- That was probably the case with Hilbert's invariants (the first controversial non-constructive proof, 1885), but definitely NOT the case with well-ordering the reals—a *single* well-ordering is asserted to exist.

Is physical existence the same as mathematical existence?

- Does the number two exist in the same sense, or in a different sense, than electrons exist?
- I took an informal poll of about twenty people, all published experts in mathematics, logic, and/or philosophy.
- The answers were extremely varied and far from unanimous.

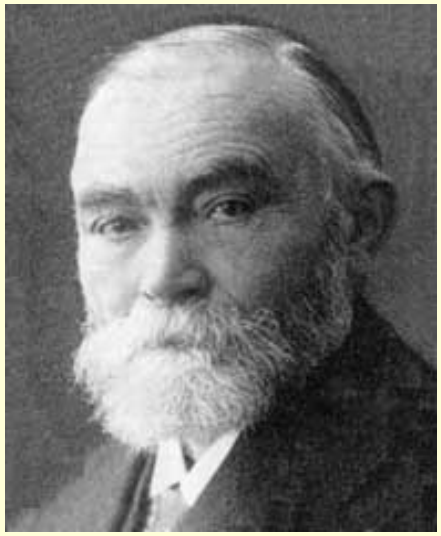
What is (the number) two?

- We directly observe pairs of things: two oranges, two apples, two grand pianos.
- These pairs have the property “two-ness” in common.
- The collection of *all* pairs has *only* the property “two-ness” in common.

The Plato-Kronecker answer

- Two is an Idea, an existing but non-physical “thing”.
- Such Ideas are “atomic”, i.e. they have no composite structure.
- Kronecker: God made the natural numbers, all the rest (of mathematics) is the work of people.

Gottlob Frege (1848-1925)



Two is the extension of the property “two-ness”

- Not the property two-ness, but the extension of that property
- The “extension” of a property means this: two properties have the same extension if all the individuals having one property have the other also.
- Frege thought that every property has an extension and these extensions are themselves individuals.

Georg Cantor (1845-1918)

- Cantor called two the “class of all pairs”.
- Extensions of properties are classes.
- Perhaps not every class is the extension of a property?
- Reification is basic to Cantor’s set theory: a set is defined (1895) as “any collection into a whole *M* of definite and separate objects *m* of our intuition or our thought.” These objects *m* are called the ‘elements’ of *M*.”

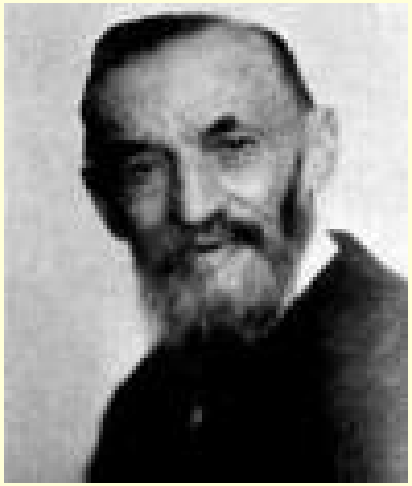
The Dedekind-Cantor-Frege answer

- TWO = reification of the property “two-ness” (of being a pair).
- “Extension” is another way of saying “reification”—considering an abstract property as a (single) object.

Reifying “dog”

- We can define
- DOG = the class of all dogs, or the extension of the concept “dog”.
- Does DOG exist? Is it as real as “two”?
- Is it the same thing as the Platonic Idea *Dog?* Dogs are dogs because they partake of the nature of *Dog.* Isn’t that about the same as “fall under the concept DOG”?

Peano gave a different answer



Guiseppe Peano
(1858-1932)

- Peano defined the natural numbers N as the least class containing 0 and closed under a unary one-to-one function s (successor), such that 0 is not the successor of anything.
- Two is then s(s(0)).

Von Neumann's formal reduction of the integers to set theory

1903-1957

- 2 is, or is represented as, {0, {0}}, where 0 is represented as the empty set.
- This picks a particular representative of each of Dedekind's classes
- Needed since classes can't be proved to exist in ZF set theory
- Not a serious suggestion that 2 really *is* this set.

Three answers to “what is two?”

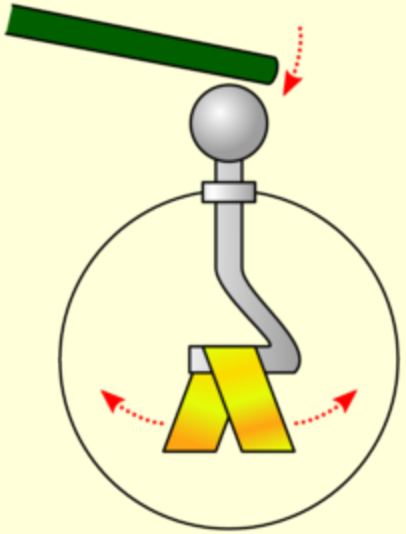
- Plato-Kronecker
- Dedekind-Cantor-Frege
- Peano

What is an electron?

- We will approach this question historically.
- What led to the discovery of electrons?
- What's the evidence for their existence?
- They say that "electron guns'' are used in the projector that is projecting this image on the screen right now. So is the fact that you can see this evidence for the existence of electrons?

An Electroscope

“Electricity” separates the gold leaves. Whatever separates the gold leaves is electricity.



Cathode Ray Tube

- A glass tube with a wire (“electrode”) embedded in each end, and most of the air removed.
- Put a charge on one electrode. You will see strange lights.

Cathode Rays

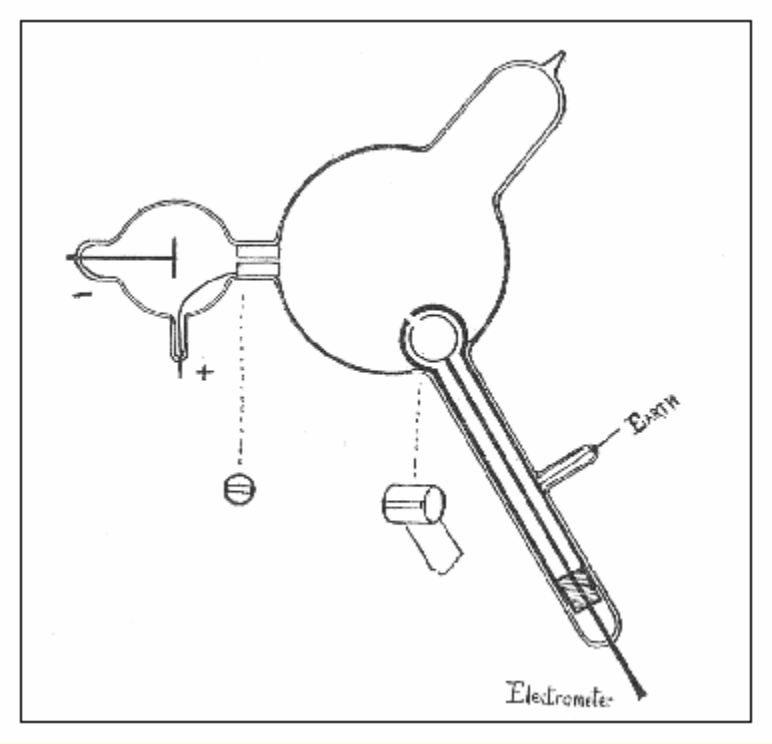
- The cause of the lights was called “cathode rays”.
- What was the nature of these rays? Were they like light rays, carried by the “aether”? Or were they some kind of particles?
- Heinrich Hertz studied this question. He built a cathode ray tube with two metal plates inside, which could be charged to generate an electric field between them. If the rays were particles, they should be deflected--but they weren't.
- So it seemed they must be waves.

J. J. Thomson (1856-1940)

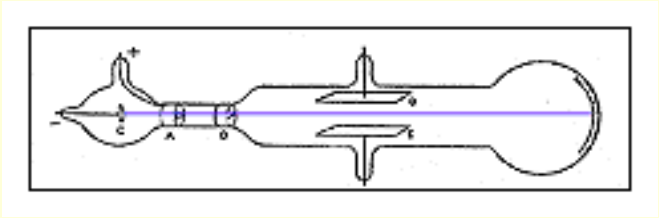
- Three key experiments (1897) demonstrated that the cathode rays were particles.
- *J.J. Thomson, "Cathode Rays,"* ***The London, Edinburgh, and Dublin Philosophical Magazine and Journal of Science****, Fifth Series, October 1897. p. 295*

Thomson's first experiment

- In the upper left a cathode ray tube produces cathode rays.
- In the large bulb they can be bent with a magnet.
- The electrometer at the lower right measures a charge if and only if the rays are bent so as to pass through a narrow slit.
- Therefore the rays carry the charge.



Thomson's second experiment

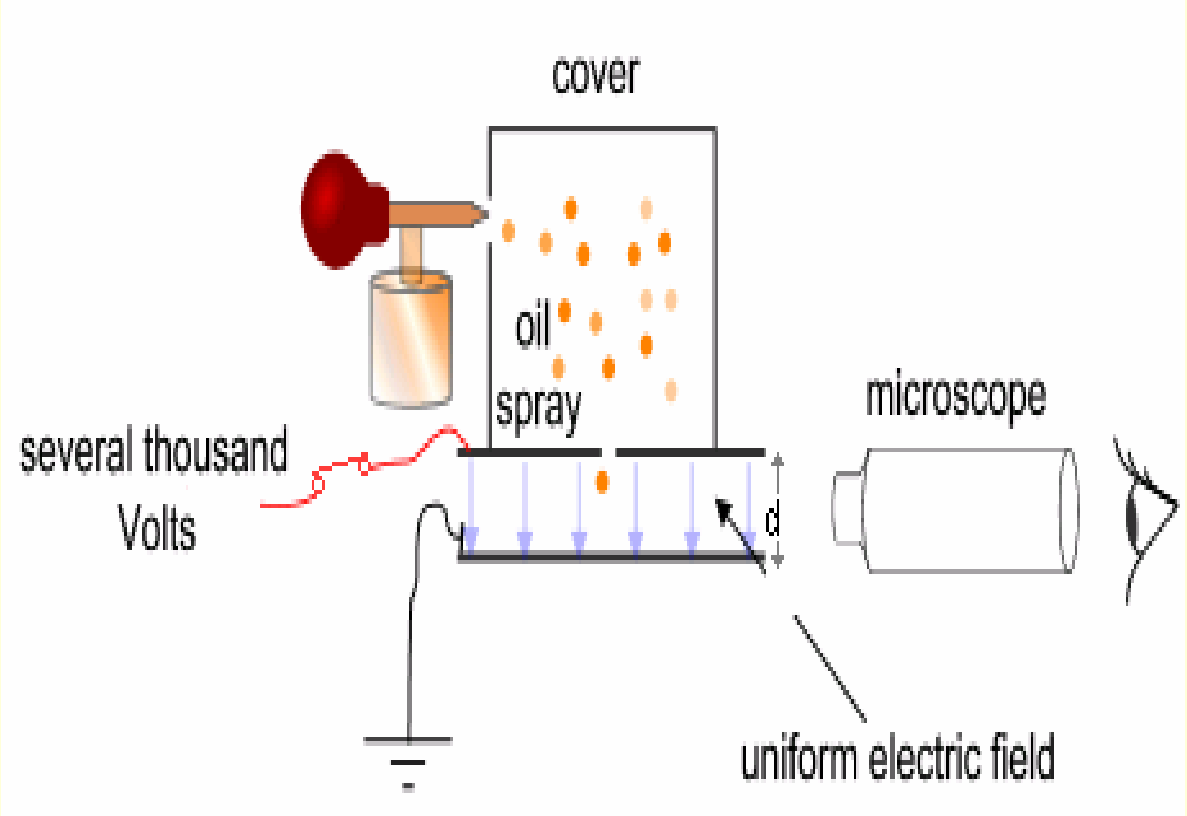


- With a better vacuum, an electric field *would* bend the rays, contrary to Hertz's result. (That was because even a small amount of gas becomes ionized and then behaves as a conductor, preventing the electric field from reaching the rays.)

Thompson's propositions

- Cathode rays are small particles carrying an electric charge
- These particles are a constituent of atoms (which were up to then considered indivisible)
- These particles are the *only* constituent of atoms (the positive charge being massless and uniformly distributed, as in a "plum pudding".)

Millikan's oil drop experiment (1911)



Robert Andrews Millikan (1868-1953)

- The small oil drops pick up one or a few electrons from the ionized medium.
- The electric force counteracts gravity and, when the electric field strength is properly adjusted, balances gravity exactly.
- That enables you to calculate the electric charge on that particular oil drop.
- They are always integer multiples of some basic “unit charge”.

Ernest Rutherford (1871-1937)

- 1911, with Geiger and Marsden.
- Bombarded gold foil with alpha particles. They expected them to "go right through".
- A small fraction bounce off at big angles.
- Conclusion: there is something small and massive inside the gold atoms. He called it the "nucleus".
- Thompson's third proposition was wrong: the positive charge is concentrated in the nucleus, and associated with mass.
- Applet animation of Rutherford's experiment

Rutherford's model of the atom

- Electrons orbit the nucleus like planets orbit the sun
- Trouble is, the laws of electromagnetism predict they should radiate away their energy, spiral into the nucleus, and the atom should collapse.

Bohr’s model of the atom

- Bohr fixed this problem by postulating that the angular momentum of an electron could only take on discrete multiples of some minimum value.
- This blocked the prediction of collapse and explained other things as well.

Wave nature of the electron

- Bohr's quantization appears less *ad hoc* if you think there is a wave associated with an orbiting electron. The orbit's circumference must be an integral number of wavelengths.
- J. J. Thompson's son George got the Nobel prize in 1937 for establishing the wave nature of the electron.
- J. J. got his Nobel prize in 1906.

Quantum Entanglement

- Today, experiments keep adding to our knowledge of electrons.
- The most recent ones cast doubt on the basic distinction between ontology and epistemology: they show that electrons do not have a well-defined spin property until the spin is measured.
- There is no time to go into this further today. But philosophers should go back to their drawing boards!

- That finishes the summary of “what two is” and “what an electron is”.
- Onwards to the poll results.
- **The question: whether two exists in the same sense as electrons or a different sense.**
- We will first just go through the answers and *then* criticize them.

Answers to the poll

- Majority of 70 or 80 percent for “different”
- The reasons given for “different” were not all consistent—some of them contradicted others.
- Roughly speaking, one-third of the respondents thought electrons are “more real” than two, one-third thought two is “more real” than electrons, and one-third thought they are “equally real”.
- Those polled later were given a chance (after answering) to critique the earlier answers. All the answers were soundly refuted.
- We’ll consider some of the answers, starting with those favoring “different”.

The Box

- “I can give you a box containing an electron, but you can’t give me a box containing two.”

Fashion in physics

"Two" is much more real than "electron".

There's a good chance that fifty years from now, the concept of electron will be obsolete—physics won't be phrased in terms of particles. On the other hand, two isn't going to change.

Interaction

You can interact with an electron, even though you can’t see or touch it, but you can’t interact with two.

“Electron” is only a concept

- The word “electron” denotes a certain concept, which we use to divide up the physical world. In reality, electrons are not separable from the rest of the universe, and really have no separate existence.

Platonism

Two is an idea, an abstraction. An electron is a tangible object.

Two is *only* an idea

There is nothing in reality that actually corresponds to that idea. There is just a property of pairs we might call “two-ness”, but it's a mistake to think that there is some *object* corresponding to that property.

An exact quote for "two is only an idea"

"Though the notion of an electron is a theoretical construct, experimental evidence massively supports the existence of 'things' behaving according to that theory. I believe that we can speak of people having the same conception of the number two, and in that sense it is an objective conception, but the evidence for this is only behavioral."

- Now for the people who answered, they exist in the *same sense.*

Only one kind of existence

- If we started admitting different “senses” or kinds of existence, we would need many kinds: one for integers, another for real numbers and sets, another for electrons, another for truth, love, justice, and beauty, another for God.
- These different kinds of “existence” would have something in common, and that something is the true meaning of “existence.”

Both are complicated concepts

- Both “two” and “electron” are concepts.
- Both are rather complicated.
- If A(x) and B(x) are two more or less equally complicated concepts, then there is no reason to think that “there exists an A” involves a different sense of existence than “there exists a B”.

Only one sense of red

- Is a cherry red in the same sense that an apple is red, or in a different sense?
- The same: there is only one sense of redness, even though red objects may differ in various ways.
- This is true even though there are different *shades* of red and the exact wavelengths constituting red may not be specified.
- Similar the meaning of “existence” might be a little fuzzy at the boundaries.

A Buddhist story

- Bodhidharma once asked Manjusri “Is the number two real?”
- Manjusri replied, “No, it is simply a thought!”
- Bodhidharma then asked “Are you real?”
- Manjusri suddenly saw through the veil that separates life and death.

- Manjusri, Bodhisattva of Wisdom, cutting through humanity's ignorance with his sword of wisdom to aid them to achieve enlightenment.

Comments on the Answers

- Each answer was criticized by at least one other later answerer.
- Well, OK, I criticized the last answer myself.

The Box

- All this shows is that two doesn't have a fixed position, while the position of an electron can be (approximately) specified.
- Your electron might have tunneled out since you put it there.
- Satan may have switched it with another electron—all of them are supposedly completely identical, whatever that means.

Fashion in physics

- It seems impossible that the concept “electron” would be shown to be contradictory, like “phlogiston”.
- Even if the electron were shown to be composite, the experiments that established its existence would still be valid.
- Besides, the concept “two” *has* changed.

Interaction

- The concept of “interaction” is itself complex and hard to explain.
- When electrons strike certain other atoms, those atoms emit photons, which interact with our retinas, which interact with our brain cells,…
- “which interact with *us*” (?)
- Now we’re doing epistemology, not ontology.
- Besides, don’t we interact with 2 every time we do arithmetic that involves 2?

Interaction

Gödel believed, as perhaps all Platonists do, that we could directly interact with mathematical objects through a faculty of our minds, something like a sense organ, but attuned to the mathematical reality rather than the physical.

“Electron” is only a concept

- Of course “electron” is a concept. That’s good—according to the Oxford Dictionary of Philosophy, we should apply “there exists” only to a concept.
- “Two” is also a concept.
- Perhaps electrons have no separate existence. Perhaps *nothing* can exist independently of everything else. But the notion of hypothetically existing independently is not under discussion.
- Why is an electron “only” a concept? How can a concept carry charge and have its motion influenced by a magnet?

Platonism (two is an idea, an electron is tangible)

- A tennis ball is tangible: we can hold it in a hand, exhibit it for others to see, point to it.
- An electron seems, by analogy, to be like a tiny tennis ball, but then it sometimes behaves quite differently, e.g. in the two-slit diffraction experiment.
- You can write your initials on a tennis ball, but you can't mark a particular electron in any way—they're all interchangeable.
- Some level of inference is required. Is that really so different from inferring the existence of two from many examples of pairs?

Two is *only* an idea

- So is my two the same as your two, or different? Would two exist if all minds disappear?
- “Electron” is a mental construction too. Thompson’s and Millikan’s experiments lead each new generation (as they pass through physics lab) to make the requisite mental constructions. But presumably electrons are not *only* ideas. So why is two different? No evidence has been advanced that two is *only* an idea.

Experimental evidence

- “There’s experimental evidence for electrons but not for 2.”
- I can’t see why performing arithmetical calculations and getting the same results as before doesn’t count as experimental evidence for the existence of numbers.

Only one kind of existence

- This argument may seem irrefutable, but it just pushes the problem under the rug, because it's the *shades* of existence that concern us today.
- We're interested in the different senses of existence, or if you say there is only one sense, we are interested in its refinements, or derived notions.

Both are complicated concepts

- This is an important observation: neither the number two nor an electron is a tangible thing like “this dog Penny”.
- But this fact doesn’t necessarily imply that “existence” has the same meaning when applied to these concepts.
- It might be so, but it would require some justification.

Only one sense of red

- But “red” is a property, and famously “existence is not a property”.
- Therefore the analogy is not correct.

The Buddhist anecdote

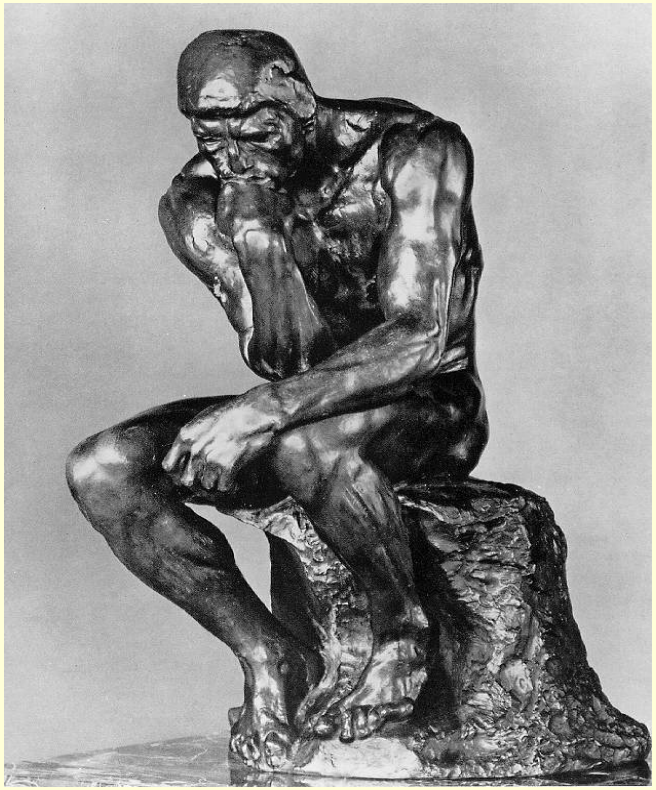
- If two is a thought, then it certainly is real, since thoughts are real.
- Perhaps Manjusri meant, in saying “two is just a thought”, that there is nothing which that thought is about, i.e. that two does not exist.
- The Buddhist notion of enlightenment is related to the idea that there is nothing which the concept of “self” is about.

The Buddhist anecdote, continued

- When one tries to extend this to electrons, one gets, “there is nothing which the concept of electrons is about”.
- But this is not just implausible, it’s false. Look back at the experiments of Thompson and Millikan. There’s *something* there, even if it also turns out to exhibit diffraction patterns, tunneling, and quantum entanglement.

That completes the discussion of the poll results

- Widespread disagreement
- After discussion still no clarity.
- Every position has some flaws.
- The situation demands philosophy.



Shades of existence

- Specific existence: This dog “Penny” exists.
- General existence: Dogs exist, ghosts exist.
- Parametric existence: Every person has (or had) a mother. Every atom contains an electron.
- Constructive existence: we can exhibit, or produce, an object falling under the concept in question.
- Further distinctions to be made soon!

Elementary Properties

- A property X is called *elementary* if we know how to recognize whether an object has property X or not. We also have to know how such objects can be *presented to us*.
- In mathematics, that means having an algorithm.
- For example, it's a dog if it has certain characteristic appearances and behaviours. This is true even though there are three-legged dogs, hairless dogs, dogs that can't play Frisbee.

Elementary properties outside mathematics

- How about, “x is a binary star at the heart of the Cat Nebula” ? Is that an elementary property?
- Yes, we know what it means to be presented with a binary star and we know how to verify its location.
- The fact that the search for one is presently impractical is not relevant.

Shades of Existence, continued

- Our theses:
- “There constructively exists an X” when X is elementary is a fundamental (primitive) notion.
- All other kinds of existence can be explained in terms of this notion by logic.

Non-constructive existence

- “there exists an X”, when X is elementary, means that it is contradictory that there should not constructively exist an X.
- If X is not elementary, proceed as with the double-negation interpretation (keep pushing the double negation inwards).
- There are plenty of examples in mathematics. To mention one: there exists a well-ordering of the real numbers.
- I will show an example in physics.

Dark Matter

- The galaxies aren't moving in the way they should be moving (according to Einstein's laws of gravity) if the visible matter is all there is.
- Therefore, there must be some invisible matter. It turns out there must be a *lot* of this stuff ("dark matter"), whatever it is.
- The concept "dark matter" refers to invisible matter that pervades the galaxies in such a way as to render their motion consistent with general relativity.

Does dark matter exist?

- Could we bottle it and put it on display in a museum, properly labeled?
- To establish the constructive existence we would have to be able to do that *and* establish that the stuff pervades the galaxies in the right densities.
- Probably once the stuff could be identified, the claim would fall into two parts: a certain kind of matter exists, *and* it has the required distribution.

Dark matter, continued

- The facts that the galaxies are far away and our lifetimes are short are not relevant to the meaning of “dark matter exists”, just as resource bounds are not relevant in constructive mathematics.
- What if we could show that it is in principle impossible to actually observe dark matter? Would it really exist?

Dark matter, continued

- Yet the existence of a well-ordering of the reals is accepted without a whimper by most mathematicians, in spite of the fact that it has been proved we shall never be able to define one in set theory.
- It did, however, raise many eyebrows a hundred years ago. This idea took some getting used to.

Non-constructive existence—an example for non-mathematicians

- Pick a well-defined question, the answer to which you don't know, such as,

  "is there a binary star at the heart of the Cat Nebula?"
- Then ask, "is there a dog, which is a Chihuahua if there is a binary star at the heart of the Cat Nebula, and otherwise is a Dalmatian?"

One dog or two?

- Most mathematicians say, yes of course: argue by cases. If there is a such a star take any Chihuahua, otherwise take any Dalmatian.
- Most other people say the question is nonsense.
- It's the "a dog" part that makes it nonsense. There are *two* dogs; we just don't know which one is the desired dog.

“There exists” in such a case is a defined concept, an abbreviation.

- It abbreviates the assertion that the constructive non-existence is contradictory.
- Classical and constructive logic give different answers, even though mathematics is not involved, because of the reasoning allowed for reaching a contradiction.
- This is a consequence of the *definition* of nonconstructive existence. It has no bearing on the meaning of constructive existence for elementary properties.

Markov's principle

- In arithmetic, elementary properties are the recursively decidable ones.
- Markov's principle says for such properties A, the meaning of existence doesn't depend on logic:
- $\neg\neg\exists x\, Ax \rightarrow \exists x\, Ax$

Status of Markov's Principle

- It isn't provable in formalized intuitionistic systems.
- It is, however, consistent with those systems; so it's not refutable.
- Russian constructivists have always believed it.
- Their argument is, we can search for *x*. The search "must terminate".

Generalized Markov's Principle

$\neg\neg\exists x\, Ax \rightarrow \exists x\, Ax$

where A is any elementary property.

For example, A(x) could be "x is a binary star at the heart of the Cat Nebula".

Status of Generalized Markov Principle (GMP)

- In the “binary star in the Cat Nebula” example, we could (in principle) search for the star—we know where to look!
- In general the “search” argument doesn’t carry over, since searching the entire universe is not as straightforward as searching the integers.
- You can’t write a computer program to do it, even if in some sense the universe might actually be finite.
- No specific instance of GMP is refutable (since the double negation is a theorem)

What about two and electrons?

- The property of being an electron is elementary.
- Is the property of being two elementary?
- Yes, we know what it means to be "presented with two": we are presented with some symbol or symbols that represent two in some standard system of notation for numbers.
- Conclusion: the right answer is "same sense".

One two or many?

- There are many electrons, but all are interchangeable and identical.
- Why do we think there is only one “two”?
- Why not many, interchangeable and identical 2’s? They are created (like electrons) and destroyed (consumed by algorithms).
- Yet these 2’s are not the same as the symbols that denote them. The symbols are used to *present* “a two” to us.

Two is more like an electron than you thought

- When physicists write equations, *e* stands for “an electron”. Any old electron!
- When we write “2 +2 = 4”, how many numbers are referenced? Two or three?
- Perhaps the property “twoness” isn’t as clear as we thought: does the set of numbers denoted by “2” in that equation have the twoness property, or not?
- 2+2 involves two “different” two’s, just as the chemical equation $H_20 = H + H + O$ refers to two different hydrogen atoms.
- We don’t normally care where those 2’s came from because all 2’s are interchangeable.

A new ontology

- TWO = class of all pairs is quite similar to DOG = class of all dogs
- Such reification is not necessary. Physicists (and even philosophers) don't need ELECTRON = class of all electrons.
- Contemporary logic demands some kind of reification to formalize mathematics.
- But perhaps it is not philosophically necessary.
- Challenge: develop a new logic that doesn't require reification to formalize mathematics.
- Or adapt an old one (linear logic? Quantum logic? Linear quantum logic?)

Conclusions

- We have analyzed the notion of existence, identifying a primitive notion: constructive existence applied to an elementary property.
- We identified the Generalized Markov Principle as a philosophical issue.
- Existence applied to non-elementary predicates is a defined concept (using logic).
- We conjecture that reification might not be necessary in mathematical ontology.
- Pending suitable formal work, that conjecture cannot yet advance to the status of a “claim”.